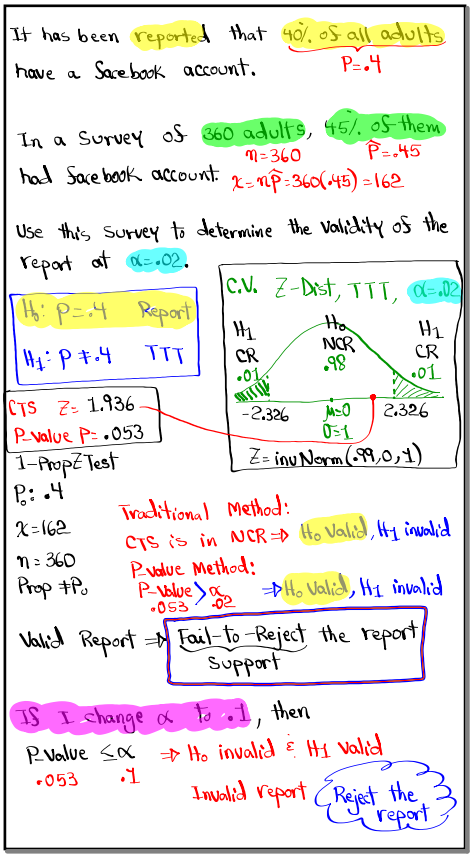
Statistics
Winter 2022
Lecture 15



The college claims that less than 10% ($<.1$) of all students are left-handed.

In a survey of 275 students ($n=275$), 8.5% of them ($\hat{P}=.085$) were left-handed.

$x=n\hat{P}=275(.085)=23.375$ → $x=24$

is decimal ⇒ Round-up

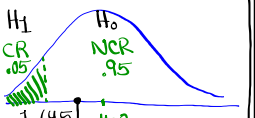
Use this survey to test the claim.

$H_0: P \geq .1$

$H_1: P < .1$ claim, LTT

CV Z-Dist, LTT

NO α → use .05



$Z=\text{invNorm}(.05,0,1)$

CTS $Z=-.704$

P-value $P=.241$

1-PropZTest

$P_0: .1$

$x=24$

$n=275$

Prop $< P_0$ LTT

CTS is in NCR ⇒ H_0 valid

P-value $> \alpha$

H_1 Invalid

Invalid claim ⇒ Reject the claim

If we change α to .25, .26, .27, .28, .29, ----

P-value $\leq \alpha$ ⇒ H_0 invalid

H_1 valid ⇒ valid claim

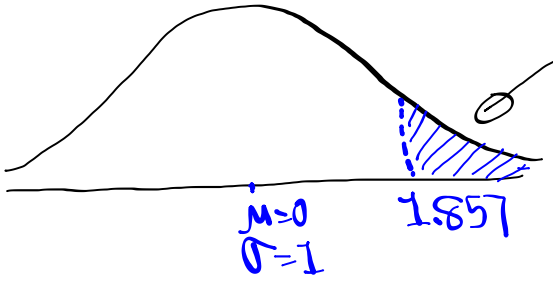
FTR the claim

What is P-value?

P-value is the area of the tail separated by the CTS.

If TTT ⇒ Multiply that area by 2.

Ex: CTS $Z=1.857$, RTT, find P-value.

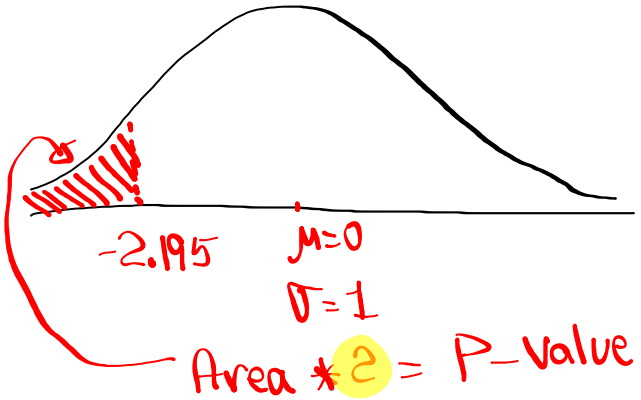


Area = P-value

= normalcdf(1.857, E99, 0, 1)

= .032

Ex: CTS $Z = -2.195$, TTT, Find p-value.



Area * 2 = P-value

$= 2 * \text{normalcdf}(-E99, -2.195, 0, 1)$

$= \boxed{.028}$

Testing One Population mean μ:

$H_0: \mu = \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu \geq \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu > \mu_0$	$H_1: \mu < \mu_0$
TTT	RTT	LTT

Case I: σ Known

CV → Z-Dist

CTS Z
P-value P → Z-Test

we proceed with testing chart to determine the validity of H_0 & H_1

Final Conclusion:

Reject the claim OR Fail-to-Reject the claim

CNN claims the mean age of all voters in the last election was 45 yrs. $\mu = 45$

I took a sample of 35 voters, their mean age was 49 yrs. $n = 35$, $\bar{x} = 49$

It is known that standard deviation of ages of all voters is 8.5 yrs. $\sigma = 8.5$

Test the claim at $\alpha = .01$.

H_0: $\mu = 45$ claim

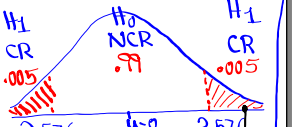
H_1: $\mu \neq 45$ TTT

σ known

CTS $z = 2.784$

P-value $P = .005$ ✓✓

CV Z-Dist TTT $\alpha = .01$



$z = \text{invNorm}(.995, 0, 1)$

Z-Test

inpt:

$\mu_0 : 45$

$\sigma = 8.5$

$\bar{x} = 49$

$n = 35$

$\mu \neq \mu_0$

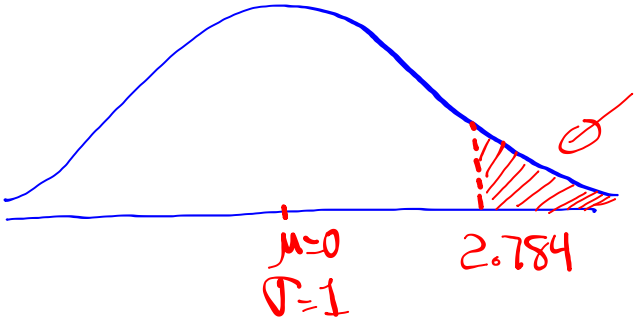
Stats

CTS is in CR ⇒ H_0 invalid

P-value $\leq \alpha$ ⇒ H_1 valid

Invalid claim ⇒ Reject the claim

CTS $z = 2.784$, TTT, find P-value.



Area $\times 2$ = P-value

2*normalcdf(2.784, E99, 0, 1)

= .005 ✓

The college bookstore claims the mean cost of all new textbooks is below $125.

$\mu < 125$

I took a sample of 30 new textbooks, their mean cost was $120.

$n = 30$

$\bar{x} = 120$

$\sigma = 15$

It is known that standard deviation of prices of all new textbooks is $15.

σ known

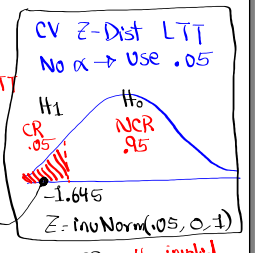
Test the claim.

$H_0: \mu \geq 125$

$H_1: \mu < 125$ claim, LTT

CV z-Dist LTT

No α → Use .05



z = invNorm(.05, 0, 1)

CTS $z = -1.826$

P-value $P = .034$

z-Test

Input: STATS

$\mu_0 = 125$

$\sigma = 15$

$\bar{x} = 120$

$n = 30$

$\mu < \mu_0$ LTT

CTS is in CR ⇒ H_0 invalid, H_1 valid

P-value $\leq \alpha$

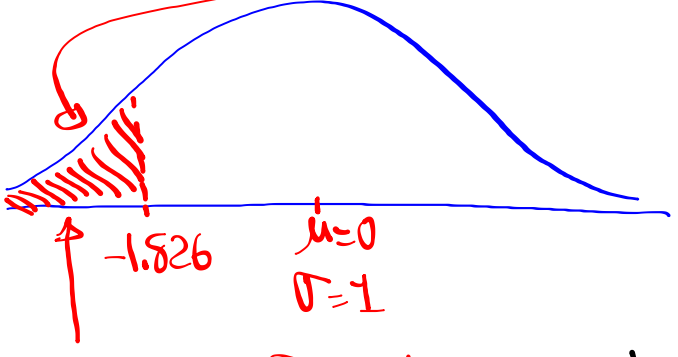
Valid claim

FTR the claim

If we change α to .02, .01

P-value .034 $> \alpha$ ⇒ H_0 valid, H_1 invalid ⇒ Invalid claim, Reject the claim

CTS $z = -1.826$, LTT, find p-value



$= .034$ ✓

Area = P-Value = normalcdf(-E99, -1.826, 0, 1)

Testing One Population mean μ:

$H_0: \mu = \mu_0$	$H_0: \mu \leq \mu_0$	$H_0: \mu \geq \mu_0$
$H_1: \mu \neq \mu_0$	$H_1: \mu > \mu_0$	$H_1: \mu < \mu_0$
TTT	RTT	LTT

Case I: σ Known	Case II: σ Unknown
CV $\rightarrow$ Z-Dist	CV $\rightarrow$ T-Dist, $df = n-1$
CTS z, P-value P $\Rightarrow$ Z-Test	CTS t, P-value P $\Rightarrow$ T-Test

we proceed with testing chart to determine the validity of H_0 & H_1

Final Conclusion:

Reject the claim OR Fail-to-Reject the claim

Given $H_0: \mu \leq 80$, claim is H_0

$\bar{x} = 84$, $S = 10$, $n = 15$, $\alpha = .02$

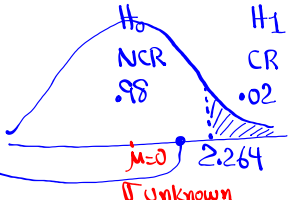
Test the claim: σ Unknown

$H_0: \mu \leq 80$ claim

$H_1: \mu > 80$ RTT

CV T-Dist RTT

$\alpha = .02$, $df = n-1 = 14$



CTS $t = 1.549$

P-value $P = .072$ ✓

T-Test

σ Unknown

$t = \text{invT}(.98, 14) = 2.264$

Inpt: STATS

$\mu_0 = 80$

$\bar{x} = 84$

$S = 10$

$n = 15$

$\mu > \mu_0$

CTS is in NCR $\Rightarrow$ H_0 Valid

P-value $> \alpha$ $\Rightarrow$ H_1 invalid

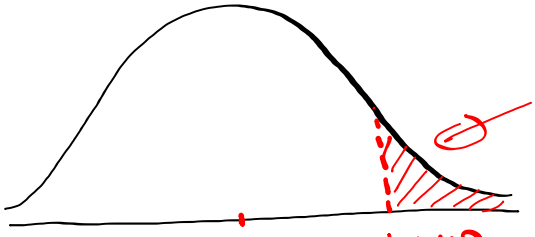
Valid claim $\Rightarrow$ FTR the claim

IS we change α to .08, .09, .1, then

P-value $\leq \alpha$ $\Rightarrow$ H_0 invalid, H_1 valid $\Rightarrow$ Invalid claim, Reject it.

P-value .072

CTS $t = 1.549$, RTT, $df = 14$, Find P-Value



Area = P-Value

$= tcdf(1.549, E99, 14)$

$= .072$

σ Unknown

I randomly selected 10 exams. Here are the results:

75	83	92	100	55
65	95	80	70	70

1) Find mean, and Standard deviation, Round to whole #. $\bar{x} = 79$, $s = 14$

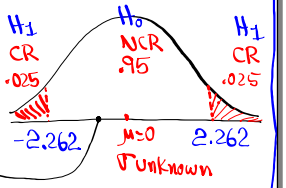
2) Test the claim that the mean Score of all exams is 80.

H_0: $\mu = 80$ claim

H_1: $\mu \neq 80$ TTT

CV T-Dist, TTT

No $\alpha \rightarrow .05$, $df = n-1 = 9$



$t = invT(.975, 9)$

CTS $t = -.226$

P-Value P = .826

T-Test

$\mu_0 = 80$

$\bar{x} = 79$

$s = 14$

$n = 10$

$\mu \neq \mu_0$ TTT

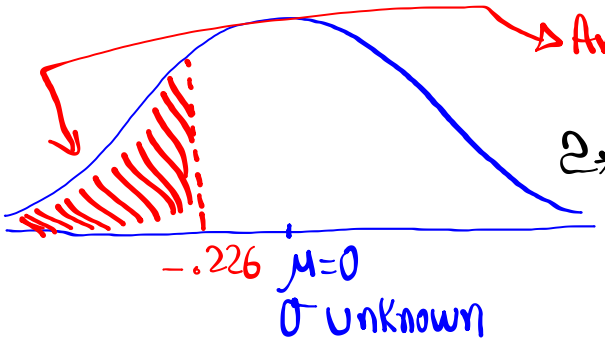
CTS is in NCR $\Rightarrow$ H_0 valid, H_1 invalid

P-Value $> \alpha$

Valid claim $\Rightarrow$ FTR the claim

CTS $t = -.226$, TTT, $df = 9$

Find P-value.



2*tcdf(-E99, -.226, 9)

= .826

LA County claims the mean age of all male police officers in the county is at most 50 Yrs. $\mu \leq 50$

I randomly selected 8 male police officers, their mean age was 55 Yrs with standard deviation of 10 Yrs. $n = 8$, $\bar{x} = 55$, $S = 10$

Use $\alpha = .1$ to test the claim.

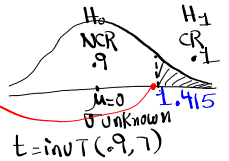
$H_0: \mu \leq 50$ claim

$H_1: \mu > 50$ RTT

σ Unknown

CV T-Dist. RTT

$\alpha = .1$ $df = n - 1 = 7$



CTS $t = 1.414$

P-value $P = .100$

T-Test

Inpt: STATS

$\mu_0 = 50$

$\bar{x} = 55$ $S = 10$ $n = 8$

$\mu > \mu_0$ RTT

$t = invT(.9, 7)$

CTS is in NCR H_0 valid

P-value $> \alpha$ $\Rightarrow$ H_1 invalid

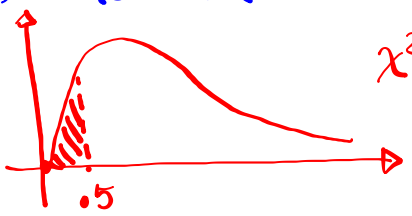
Valid claim $\Rightarrow$ FTR the claim

If we change α to .11, .12, .13, .2, .3, ...

H_0 invalid $\Rightarrow$ Reject the claim

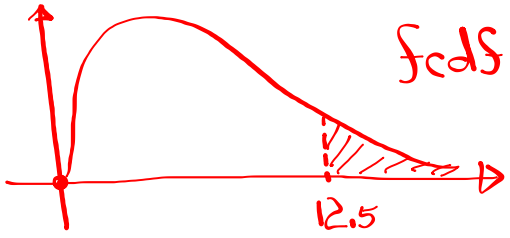
Extra Credit QZ (Added to exam Score)

1) Find $P(\chi^2 < .5)$ with $df = 9$.



$\chi^2cdf(0, .5, 9) = \boxed{3.04 \times 10^{-5}}$

2) Find $P(F > 12.5)$ with $Ndf = 5$, $Ddf = 30$.



$Fcdf(12.5, E99, 5, 30)$

$= \boxed{1.3 \times 10^{-6}}$